MATH5010 Linear Analysis (2022-23): Homework 7. Deadline: 27 Nov 2022

Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard.

1. Let $(X, \langle \cdot, \cdot \rangle)$ be an inner product space. Show that the inner product $\langle \cdot, \cdot \rangle : X \times X \longrightarrow \mathbb{C}$ is continuous, that is, whenever the sequences $x_n \to x$ and $y_n \to y$ in X, we have $\langle x_n, y_n \rangle \to \langle x, y \rangle$.

2. Let $(X, \langle \cdot, \cdot \rangle_X)$ and $(Y, \langle \cdot, \cdot \rangle_Y)$ be Hilbert spaces. For $(x_1, y_1), (x_2, y_2) \in X \times Y$, put

$$\langle (x_1, y_1), (x_2, y_2) \rangle_{X \times Y} := \langle x_1, x_2 \rangle_X + \langle y_1, y_2 \rangle_Y.$$

   Show that $\langle \cdot, \cdot \rangle_{X \times Y}$ is an inner product on the direct sum $X \times Y$ and it is a Hilbert space under this inner product.

$* * *$ **End** $* * *$